An effective theory for conductance by symmetry breaking

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Abstract

An effective theory is suggested for the particle-anti particle and the particle-particle modes of strongly disordered electron systems. The effective theory is studied in the framework of the saddle point expansion and found to support a vacuum which is not invariant under translations in imaginary time. The Goldstone bosons of this symmetry breaking generate a pole in the density-density correlation function. The condensate of the auxiliary field corresponding to the particle-particle channel produces conductivity without relying on the long range fluctuations. The results are obtained for $d > 1$.

I. INTRODUCTION

The understanding of the conductor-insulator transition of strongly disordered systems represents a challenging problem due to the large number of effects which may come into the scene. We approach this problem in this paper by means of an effective theory description. Such a method is slightly better known and more widely used in the High Energy Physics community but we hope that such a different point of view may lead to a more complete picture.

The description of the influence of local impurities on the collective phenomena started with the one particle approximation. The presence of the periodic crystal and the electron-ion interaction yields a model for the Bloch-Wilson band insulators [1] and for the Peierls transition [2], the difference being the dynamical origin of the gap opening at the Fermi surface. As the disorder increases such a description becomes inapplicable and one arrives

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at the disorder induced transition to Anderson localization [3], understood qualitatively by simple scaling arguments [4]. In addition to the disorder, the electron-electron interaction, as well, was thought to be responsible for the Mott transitions [5] where the gap originates either in the long range order of the moments or in the quantum phase transition induced by charge correlations.

After these first intuitive steps more systematic approaches were developed along two different lines, based on the partial resummation of the perturbation expansion and the non-perturbative use of effective models.

The Schwinger-Dyson resummation of the one-loop self energy for the electron propagator or the Bethe-Salpeter resummation of the one-loop ladder exchange reproduced the finiteness of the lifetime of a charge on a random, static impurity background and the classical conductivity expressions [6]- [8]. The crossed exchange contributions to the conductivity [9]- [11] and the cancellation of the diffusion pole [12] underlined the similarity between the conductance in strongly disordered systems, and the long range modes in the particle-particle scattering process. Numerous works were devoted to the introduction of the electron-electron interaction [13] to make the partially resummed perturbative description more realistic.

In the other approach different effective theories are used for the strongly disordered systems which are based on the self-consistent mode-coupling and operate with effective fields responsible for the electron propagation [14]- [17]. The dynamical origin of these matrix fields was found in the imaginary time formalism [18]- [20] where the matrix index is provided by the Matsubara frequency in the energy space.

There are several examples where the sudden appearance of conductivity, as the disorder is decreased, is related to the dynamical breakdown of the time reversal invariance. This is obvious for weak localization where the loss of constructive interference for the time reversed trajectories removes the driving force to localization. The descriptions based on effective models offer another mechanism by providing a continuous symmetry whenever the time reversal invariance is intact. In fact, the pole in the current correlation function is generated in a manner reminiscent of the Goldstone theorem [16]. The continuous symmetry in question is the mixing of the fields responsible for the retarded and the advanced propagators. When the matrix valued fields in the Euclidean energy space are used [18]- [20] then the retarded and the advanced propagators are related to the positive and the negative Matsubara frequency components of some fields. The continuous symmetry which is broken dynamically in the conducting phase is the mixing of the positive and the negative Matsubara frequency modes. It is a special rotation in the space of the Matsubara modes which flips the sign of the frequency and exchanges the retarded and the advanced propagators: the time reversal transformation.

There are two features of these effective theories which one finds unsatisfactory:

- The mixing of the positive and the negative Matsubara frequency modes is an **approximate symmetry** only because the time derivative piece of the action is clearly non-symmetrical. Even though this term should be irrelevant at the phase transition it would be preferable to have an exact formal symmetry whose breakdown is the dynamical origin of the phase transition. The origin of the exact symmetry is hidden by the use of the Matsubara frequency space. The circumstance that the dimension of the Matsubara frequency space is proportional to the UV cutoff in time suggests

that the symmetry in question is actually a time dependent gauge symmetry. In fact, the time dependent and space independent gauge transformations mix the Matsubara modes and the time derivative term in the action is the only one which breaks the symmetry with respect to them. The spontaneous breakdown of the symmetry with respect to a global subgroup of a gauge group has already been successfully treated in space-time, avoiding the use of the momentum or the frequency space [21]. Thus it is more natural to recast the effective theories [18]- [20] by using the time rather than the frequency variable.

- Another difficulty related to the use of the frequency space is the loss of the control over **non-locality** of the effective theory. The models [18]- [20] are based on bilocal fields and even if the saddle point is localised the fluctuations around it correspond to a genuinely non-local model. In lacking of the general theory of the non-local field theories the reliability of the loop expansion around such a non-local background field is not known and the apparent simplicity of the one-loop solution is misleading.

The cure of both problems is the use of time rather than frequency. We present in this paper a simple effective theory for the phase transition corresponding to the appearance of conductance by the dynamical breakdown[1] of gauge and time inversion symmetry in space-time.

Our basic assumption is that the direct effects of quenched disorder on the electrons can be incorporated in an effective theory with quasi-local interactions in time. Thus the modification of the program of refs. [18]- [20] consists of the use of local composite fields in time instead of the matrix valued fields and the generation of the non-locality by quasi-local, higher order time derivative terms in the action. The expected non-locality together with the breakdown of the gauge and the time reversal invariance will be generated in the framework of the saddle point expansion by a dynamical symmetry breaking. The non-locality in time will originate from the presence of a "condensate", a non-trivial saddle point in time. It will be shown that a finite number of higher order derivatives in time is sufficient to generate the desired symmetry breaking pattern. This description, kept in space-time, is simpler than the one given in frequency space. Another bonus of keeping the time in the description is the fact that the time dependent saddle point breaks an exact continuous symmetry, the time translation symmetry which here provides a mechanism to generate conductivity.

The important pieces of the effective theory to generate the conducting phase, the higher order derivative terms in time are irrelevant in the disordered,localized phase thus their coupling constants are not yet constrained by experimental data collected in the localised phase. We do not derive the couplings either, only show that, as they depend on the environmental parameters, such as the Fermi energy of the electrons, they can drive the phase transition to delocalisation without playing an important role in the localized phase. The justification of our choice of the effective coupling constants, which we leave for a later work, can be addressed by means of the current methods in Quantum Field Theory [22]. All

[1]The dynamical breakdown of a symmetry differs from the spontaneous one in what it occurs at finite instead of zero energy, and is identified by an order parameter with finite length or time scales as opposed to the homogeneous order parameter of the spontaneous symmetry breaking.

what is needed is a computation of the connected or the one particle irreducible (1PI) graphs with high accuracy in their dependence on the external momenta in the infrared regime.

We restrict ourselves to the discussion of the effects of quenched impurities, ignoring the Coulomb interaction. A more reliable description naturally requires more sophisticated methods, such as the renormalization group treatment [19], [22]. The failure or the success of these methods devised for local theories will be the measure of the importance of keeping the effective theory local.

The organization of the paper is the following. Our effective theory is introduced in Section II for the diffuson and the cooperon auxiliary fields. The symmetry breaking mechanism with inhomogeneous vacuum is briefly introduced in Section III. The effective model with diffusons and cooperons is discussed in Sections IV and V, respectively. Section VI is a summary of the work and the results.

II. EFFECTIVE THEORIES

One of the most important effects to reproduce in an effective theory is the appearance of non-local structures, such as composite particles or condensates. If the interaction generated by the elimination of certain modes is of short range and attractive then new extended bound states may appear. If this interaction is of long range then condensation and spontaneous symmetry breaking may take place. We believe that the quenched disorder which is usually taken into account by some auxiliary field does not generate long-range interaction in space. Instead, the dynamical symmetry breaking happening in the time direction suggests that the static nature of the impurities leads to long-range interactions in time which must be preserved in the effective description. Our goal is to find an effective action which incorporates in an economical manner this feature which we believe to be the key to the interactions generated by the quenched impurities.

Let us start with a remark about the distinction of the eventual non-locality of the interaction vertices in the action from those of the interactions generated by it. The locality of an action can be classified by considering the distance r in wich the field variables are coupled in units of the UV cutoff. The potential term without gradient is ultra-local, $r = 0$. The models with derivatives up to a finite order, $0 < r < \infty$ are quasi-local. Finally the non-local theories are with gradients of infinite order, $r = \infty$. The relation between this parameter r of certain terms in the action and their contribution to the correlation length ξ (in units of the cutoff) reflects the relevance of the terms in question. In fact, an irrelevance of a higher order derivative term with dimensionless coupling constant g implies $d\xi/dg \to 0$ as the cutoff tends to infinite. It is worthwhile noting that an effective interaction which arises from the elimination of some local degrees of freedom is always formally non-local. This happens because those contributions to the 1PI (1 particle irreducible) functions where the internal lines correspond to the eliminated modes contribute to the effective coupling constants induced by the blocking and they display non-polynomial momentum dependence. The corresponding blocked action, the generating functional, is non-polynomial in the gradient. The important question is whether the infinite series in the gradient can be truncated at a finite order without modifying the universality class of the model. If this can be done the effective theory can safely be classified as quasi-local.

The issue of non-locality of the interactions arises at the IR end point, when all modes are eliminated and the true 1PI functions are considered. For the short range interactions they are infrared finite. The Taylor expansion of the 1PI graphs in the momenta is well defined at $p = 0$ and the generating functional, the effective action is quasi-local. In the presence of long range interactions there is an IR instability, the expansion in the momentum is ill defined at $p = 0$, the effective action is non-local. Note that the only way we know for a massive, non-critical effective actions to be non-local is the condensation or the spontaneous or dynamical symmetry breaking. Then the long range interactions indicate that the true vacuum of the theory is different from the one around which the gradient expansion of effective action was carried out. In fact, the effective action of a non-critical theory will always be quasi-local when its field variable corresponds to the fluctuations around the true vacuum.

The formal indication of long range interactions for the charged particles in the presence of quenched impurities is that the mean-field of the models [18]- [20] is a bilocal field which is proportional to the sign of the Matsubara frequency. The mean field generation is not a spontaneous symmetry breaking, a phenomenon driven by the infrared modes and leading to homogeneous condensate corresponding to a local field variable. Instead the non-trivial frequency dependence of the mean-filed suggests a dynamical symmetry breaking generated by the time derivative term in the action. The effective theory sought should

- be **quasi-local** at the cutoff because the relevant part of the interactions are well known in the UV region, leaving the irrelevant, higher derivative, radiative correction part open for phenomenological adjustments, and

- contain the **long range interactions** in the IR sector accounting for the condensate with this particular time dependence.

The solution is the use of higher order derivative terms in time which are irrelevant on trivial or on homogeneous background but can change the universality class when the vacuum is inhomogeneous [23], [24]. In order to keep track of the issue of locality the effective action will be constructed in time instead of frequency space.

This program consists of the construction of a chain of effective theories. We follow the general approach leading to the effective theories of refs. [18]- [20] except that the auxiliary composite fields, responsible for the particle-hole and the particle-particle channels will be kept local instead of bilocal. Such a limitation in the treatment of non-local effects will be compensated for by retaining the terms in the effective action which are higher order in the time derivative.

The first level is for the electrons and the photons, represented by the field ψ and the time component of the photon field u, respectively,

$$Z = \int D[\psi^*]D[\psi]D[u]e^{\frac{i}{\hbar}S[\psi^*,\psi,u]}, \tag{1}$$

after having eliminated the quenched impurities by means of the replica method. In order to accommodate electrons with finite density the chemical potential μ of the electrons should be introduced in S. The effective coupling constants in $S[\psi^*, \psi, u]$, i.e. any interaction vertex beyond the minimal coupling originate from the interactions between the electrons and the impurities. This effective theory is supposed to be obtained by means of a perturbation

expansion whose small parameter is an effective electron-impurity coupling constant, denoted by g. The Coulomb interaction is kept explicitly in the model by retaining the field u. The reason for eliminating the impurities but keeping photons is that the condensation of charged particles usually induces a mean field, i.e. condensate for the photon, as well, a phenomenon which is difficult to reproduce once the photons have already been eliminated perturbatively. Thus we prefer to keep the photon field at hand to reproduce later the condensation in the framework of the saddle point expansion if needed. The photon dynamics is kept on the tree-level in this paper, the inclusion of the Coulomb interaction between the electrons will be ignored. According to the general remarks about non-locality in time the most important pieces of $S[\psi^*, \psi, u]$ for our purpose are the terms with higher order derivatives in time.

The influence of quenched impurities on the electrons is usually taken into account by a static potential, averaged after the quantum expectation values have already been computed [7]. The electron propagation is influenced for an arbitrary long time by each static, random potential. Such a long time modification survives the averaging over the different realizations of the potential. One can understand this by recalling that the elimination of a particle always generates non-local interactions. The range of this interaction is the correlation length which is supposed to be generated by the interaction with the particle in question. Since the static potential is of infinitely long range in time its elimination produces long time correlations. On a more formal level, the static potential, $V(x)$ multiplies the time integral $I(x) = \int dt \psi^*(t,x)\psi(t,x)$ and its fluctuations generate the terms $I^n(x)$, $n = 2, 3, \cdots$ in the effective action which are highly non-local in time. This non-locality leads to the rather singular

$$\Lambda(t_1, t_2) \approx \sum_{n>0}^{N} \sin(t_1 - t_2)\pi(2n+1)T \tag{2}$$

bilocal mean-field of ref. [18]- [20], where T is the temperature. The non-triviality of this field configuration is due to the non-locality confined at the ultraviolet cutoff scale, $t_1 - t_2 \approx 1/N$. The extended structure within this short time interval generates higher order derivatives for the electron field in the effective action, a generic term for two operators $A(t)$ and $B(t)$ being

$$\int dt_1 dt_2 A(t_1)\Lambda(t_1 - t_2)B(t_2) = \sum_{\ell=0}^{\infty} \frac{G_\ell}{\ell!} \int dt A(t)\partial_t^\ell B(t), \tag{3}$$

where the coupling constant

$$G_\ell = \int dt \Lambda(t)(-t)^\ell \approx \sum_{n>0}^{N} \frac{1}{(n\pi T)^{n+1}} \int_0^{2n\pi} ds(-s)^n \sin s. \tag{4}$$

The contribution in the last expression of a given n is $\mathcal{O}(n!/n^{n+1})$ which gives a rapidly converging series. Thus there will be few, cutoff independent, higher order derivative terms in the effective action. This heuristic argument is to explain the relation between the mean-field (2) and the appearence of the higher order derivatives only. Instead of a more precise derivation of these effective coupling constants in the effective theory (1) we proceed with a more phenomenological approach and consider two simple cases only.

Anticipating the importance of composite quasiparticles we introduce a $Q^{j,k}_{\alpha,\beta}(x,t)$ charged cooperon and a $N^{j,k}_{\alpha,\beta}(x,t)$ ($N^\dagger=N$) neutral diffuson field, where α,β and j,k are the spin and the replica indices, respectively, and approximate (1) by

$$Z=\int D[\psi^*]D[\psi]D[Q^*]D[Q]D[N]D[u]e^{\frac{i}{\hbar}(S_C[u]+S_\psi[\psi^*,\psi,Q^*,Q,N,u]+\tilde{S}_{Q,N}[Q^*,Q,N,u])} \tag{5}$$

where

$$\begin{aligned}
S_C&=\int dtdx\frac{1}{2}(\nabla u)^2,\\
S_\psi&=\int dtdx\Big\{Z_\psi\psi^{j*}_\alpha K_\psi(i\hbar D_{\psi,t})\psi^j_\alpha-\frac{\hbar^2}{2m}\psi^{j*}_\alpha\Delta\psi^j_\alpha-\mu\psi^{*j}_\alpha\psi^{*j}_\alpha\\
&\quad+\psi^{*j}_\alpha\psi^{*k}_\beta Q^{k,j}_{\beta,\alpha}+Q^{k,j*}_{\beta,\alpha}\psi^j_\alpha\psi^k_\beta+\psi^{j*}_\alpha N^{j,k}_{\alpha,\beta}\psi^k_\beta\Big\},\\
\tilde{S}_{Q,N}&=\int dtdx\Big\{tr\Big[\tilde{Z}_Q Q^\dagger\tilde{K}_Q(i\hbar D_{Q,t})Q-\frac{\hbar^2}{2\tilde{M}_Q}Q^\dagger\Delta Q\\
&\quad+\tilde{Z}_N N\tilde{K}_N(i\hbar\partial_t)N-\frac{\hbar^2}{2\tilde{M}_N}N\Delta N\Big]+\tilde{U}\Big\},
\end{aligned} \tag{6}$$

and

$$D_{\psi,t}=\partial_t-i\frac{e}{\hbar c}u,\qquad D_{Q,t}=\partial_t-2i\frac{e}{\hbar c}u. \tag{7}$$

This effective theory is constructed in the spirit of the Landau-Ginzburg double expansion in the amplitude and the gradient of the field. The original effective theory (1) contained higher order derivative terms both in time and the space. We believe that the non-locality in space is not essential in the problem considered and we keep the terms $\mathcal{O}(\Delta)$ only. We expect the photon dynamics to be left untouched by the localisation thus the u dependence has to be retained in the leading order and the higher order derivatives terms for the photon field are ignored.

According to the gradient expansion the coefficient functions of the retained derivative pieces depend on the local field variables, except u. The functions

$$\tilde{Z}_\psi(Q^*,Q,N)=1+O(g),\quad \tilde{Z}_Q(Q^*,Q,N)=O(g),\quad \tilde{Z}_N(Q^*,Q,N)=O(g), \tag{8}$$

and

$$K_\psi(z)=z+O(g),\quad \tilde{K}_Q(z)=z+O(g),\quad \tilde{K}_N(z)=z+O(g), \tag{9}$$

control the strength and the structure of the correlations in time. These functions will be chosen to be polynomials of finite order in order to preserve the quasi-locality at the cutoff of this effective theory. Note that the time reversal invariance of the original, microscopic dynamics renders the lagrangian real, in particular

$$K(z)=\sum_{n=1}^{M}c_nz^n, \tag{10}$$

where the coefficients c_n are real. The kinetic energy contains the effective masses

$$m(Q^*,Q,N)=m+O(g),\quad \tilde{M}_Q^{-1}(Q^*,Q,N)=O(g),\quad \tilde{M}_N^{-1}(Q^*,Q,N)=O(g). \tag{11}$$

$\tilde{Z}_Q\neq 0$ or $\tilde{Z}_N\neq 0$ indicates the presence of bound states with diffusion constant $\hbar^2\tilde{M}_Q^{-1}$, $\hbar^2\tilde{M}_N^{-1}$ in the appropriate channels in the absence of the gap when the fluctuations controled by the kinetic energy are of long range. The potential $\tilde{U}(Q^*,Q,N)$ comprises the ultra-local interactions between the composite particles. The rotational invariance restricts the appearance of the fields Q^*, Q, and N in these functions to the combination $tr(Q^*\sigma_2)^k(\sigma_2 Q)^\ell N^m$, where σ_j denotes the Pauli matrices. The functions above can be obtained by following the standard procedure of the determination of effective theories [25].

Since the long range fluctuations correspond to cooperon or diffuson fields the electrons should be eliminated,

$$Z=\int D[Q^*]D[Q]D[N]D[u]e^{\frac{i}{\hbar}S_{eff}[Q^*,Q,N,u]} \tag{12}$$

where

$$S_{eff}=\frac{1}{2}trlog\begin{pmatrix} Q^* & Z_\psi K_\psi-\frac{\hbar^2}{2m}\Delta-\mu+N\\ Z_\psi K_\psi-\frac{\hbar^2}{2m}\Delta-\mu+N & Q\end{pmatrix} \tag{13}$$

$$+S_C[u]+\tilde{S}_{Q,N}[Q^*,Q,N,u]. \tag{14}$$

By following the same approximation for the fermion determinant as (6) was obtained we arrive at the form

$$S_{eff}[Q^*,Q,N,u]=S_C[u]+\int dtdx\Big\{tr\Big[Z_Q Q^\dagger K_Q(i\hbar D_{Q,t})Q-\frac{\hbar^2}{2M_Q}Q^\dagger\Delta Q$$
$$+Z_N N K_N(i\hbar\partial_t)N-\frac{\hbar^2}{2M_Q}N\Delta N\Big]+U\Big\}. \tag{15}$$

One could have assumed the form (15) as our starting point and have applied the arguments leading to (8)-(11) to obtain directly (15) instead of following the sequence (1)→(5)→(15). The only additional insight one gains from longer the path is that the auxiliary fields Q^*, Q and N decouple from the electron field ψ in the weak disorder limit, $g\to 0$, and a highly singular effective action is found,

$$e^{\frac{i}{\hbar}\tilde{S}_{Q,N}[Q^*,Q,N]}\to\delta(Q^*)\delta(Q)\delta(N), \tag{16}$$

to suppress their fluctuations. This problem can be avoided by the introduction of an additional auxiliary field as in ref. [20]. Since we are interested in the dynamics around the phase transition when the disorder is strong we keep our presentation on the simpler level.

As mentioned above, the effective theory (15) differs from the one proposed in refs. [18]-[20] by including local fields but there are higher order derivative terms. The net result of using a quasi-local theory is that the saddle point expansion which generates the necessary non-local effects is under control because the higher loop contributions remain local, as opposed to the models with bilocal fields.

It is worthwhile to note the formal similarities between the localization and the quark confinement. Both are related to the same infrared instability, the mass gap generation mechanism and can easily be obtained in local expansions. The haaron-model for the vacuum of the gluonic sector offers a view of the confinement as an Anderson localization in 4+1 dimensions [26]. Both mechanisms are effective at low energies, at high temperature the quarks deconfine, the electrons delocalize. The deconfinement mechanism is related to states with non-zero coherence length in time which is realized by the condensate of the gluon field. We shall argue below that the transition of a strongly disordered system to the conducting phase might be generated by the coherent states which support the phase coherence in time. The resulting finite conductivity differs substantially from the classical one, arising from the incoherent scatterings.

III. SYMMETRY BREAKING

The effective theory (12) with computable but yet unknown coefficient functions is supposed to generate a long range interaction in time. Our goal is to show that a suitable chosen quasi-local effective theory generates, after a dynamical symmetry breaking is taken into account, highly non-local interactions.

The key feature, the non-locality is reflected in the higher order derivatives and has two important effects. One which comes from the perturbation expansion is the appearance of additional poles on the complex frequency plane of the loop integrals [30]. When these poles are at complex energies then they make the lifetime finite. The imaginary part of the energy at the pole casts doubt on the consistency, the unitarity, of the model. Perturbation expansion can be used to argue that the imaginary contributions of the complex poles which always come into complex conjugate pairs cancel in the optical theorem at sufficiently low energy [31] and such theories can be consistent [32]. Problems may arise at the non-perturbative level due to the lack of convergent path integral for the negative norm states corresponding to the complex poles [33] but reflection positivity [34] and the existence of a subspace with positive definite metric and unitary time evolution can be proven for a large class of models [23].

Another, non-perturbative effect of the higher order derivative terms is that they may generate non-homogeneous vacuum and new critical behavior [23], [24]. The inhomogeneous vacuum in time would easily be spotted experimentally by the non-conservation of the energy. Thus the experimentally well confirmed energy conservation excludes the time dependence of the vacuum. But the situation changes when the $t \to t + i\tau$ analytic continuation for imaginary time is considered. In fact, the breakdown of the translation symmetry of the vacuum in the imaginary time direction does not imply non-conservation of the energy for real time processes. In particular, it will be checked that the vacuum remains time independent in real time in the coupling constant range considered. The saddle point which depends on $\mathrm{Im}(t)$ is not an analytical function of t thus the Wick rotation becomes rather non-trivial, a complication whenever the saddle points are available for imaginary time only. We use in the present work the evolution in imaginary time as a projection onto the vacuum leaving the issue of the detailed Wick rotation for subsequent work. Our expressions refer to a well defined finite Euclidean time interval τ but in order to find the ground state properties we shall always be interested in the limit $\tau \to \infty$.

It will be shown that the saddle point of the Euclidean effective theory (12) can be inhomogeneous,

$$Q(x,\tau) = \chi(x)e^{i\omega\tau}, \quad N(x,\tau) = N_0 \cos\omega\tau. \tag{17}$$

According to the by now standard semiclassical expansion [35] the inhomogeneity of the vacuum gives rise to the dynamical breakdown of the symmetry with respect to the translations of the imaginary time, generates Goldstone modes and a pole in the density-density correlation functions in the leading order of the saddle point expansion. The remarkable effect of the inhomogeneous vacuum in imaginary time is that some localized electron state may become delocalized even at the tree level. Recall that the single particle states for a given time are easiest to obtain in the Euclidean theory and the localization is the result of a destructive interference of the potential barriers, the capture of the electron states in the random, static potential valleys. The (imaginary) time dependence of $Q(x,\tau)$ not only breaks the time inversion symmetry but allows the electrons to borrow (Euclidean) energy in units of $\hbar\omega$ from the vacuum to escape the potential valleys and thereby tends to delocalize the system. Taking into account the inhomogeneous vacuum of the appropriate effective theory in the framework of the saddle point expansion one finds a systematic approach to problems like the formation of the solid state crystal, the appearance of the rotons and the periodic ground state for He^4, and finally the dynamic origin of the charge density phase.

We do not attempt to derive here the effective action (12) from an underlying microscopic theory. Instead, we shall be satisfied to show that for a certain choice of the effective action the inhomogeneous vacuum (17) is formed and the density-density or the current-current correlation functions contain a pole and become long-ranged. The latter case is interpreted as the sign of the conducting transition in the framework of this effective theory.

IV. DIFFUSONS

The diffuson denotes a collective mode in the perturbation expansion which produces an infrared pole in the propagator of the composite operator $\psi^*\psi \approx N$. We assume that there are propagating electron-hole states, i.e. $Z_N \neq 0$ and there is a linear term in $K_N(z)$. After the rescaling $N \to \sqrt{Z_N c_1}N$ the effective action $S_{Q,N}[Q^* = 0, Q = 0, N, u = 0]$ is assumed to contain $Z_N = 1$, $M_N = M$, and

$$\begin{aligned} K_N(z) &= z + c_2 z^2 + c_3 z^3 + c_4 z^4, \\ U_N(N) &= \frac{1}{2}G_1 trN^2 + \frac{1}{2}G_2(trN)^2 + \frac{1}{2}G_3(trN^2)^2, \end{aligned} \tag{18}$$

where $G_3 > 0$,

$$L_N = tr\left[N(i\hbar\partial_t - c_2\hbar^2\partial_t^2 - c_3 i\hbar^3\partial_t^3 + c_4\hbar^4\partial_t^4)N - \frac{\hbar^2}{2M}N\Delta N\right] + U_N(N). \tag{19}$$

The truncation of the potentially infinite order polynomial in (18) will be justified later.

The Wick rotation to imaginary time yields

$$L_N^E = tr\left[N(\hbar\partial_\tau + c_2\hbar^2\partial_\tau^2 + c_3\hbar^3\partial_\tau^3 + c_4\hbar^4\partial_\tau^4)N - \frac{\hbar^2}{2M}N\Delta N\right] + U_N(N) \tag{20}$$

where the stability of the vacuum requires the highest order non-vanishing coefficient, c_4, be positive. The path integral

$$Z = \int D[N] e^{-\frac{1}{\hbar} S^E[N]} \tag{21}$$

is saturated in the loop expansion by the configuration which minimizes $Re S^E$ and cancels the derivative of $Im S^E$. A necessary condition for this is the equation of motion,

$$[\hbar\partial_\tau + c_2\hbar^2\partial_\tau^2 + c_3\hbar^3\partial_\tau^3 + c_4\hbar^4\partial_\tau^4]N - \frac{\hbar^2}{2M}\Delta N + (G_1 + G_3 tr N^2)N + G_2 tr N = 0. \tag{22}$$

Let us look for the solution of the form

$$N^{j,k}_{\alpha,\beta} = \delta^{j,k}(n_0(\tau) + n(\tau)\cdot\sigma)_{\alpha,\beta}, \tag{23}$$

where n_μ $\mu = 0, \cdots, 3$ are real and σ^j are the Pauli matrices. The equation of motion is

$$0 = [\hbar\partial_\tau + c_2\hbar^2\partial_\tau^2 + c_3\hbar^3\partial_\tau^3 + c_4\hbar^4\partial_\tau^4 + G_1 + 2G_3 n^2 + 2G_2\delta_{\mu,0}]n_\mu, \tag{24}$$

where $n^2 = n_\nu n_\nu$. The solution wil further be simplifed by assuming the form

$$n_\mu(\tau) = n_\mu \cos\omega\tau, \tag{25}$$

n_μ being a constant four vector. We obtain two separate equations since the odd and the even order time derivatives generate sines and cosines, respectively. The coefficients of $\sin\omega\tau$ cancel if

$$\omega^2 = \frac{1}{\hbar^2 c_3}. \tag{26}$$

The equation for the terms with $\cos\omega t$ is

$$n_j n_j = \frac{(c_2 - c_4\hbar^2\omega^2)\hbar^2\omega^2 - G_1 - 2G_2}{2G_3}, \quad n_0 = 0, \tag{27}$$

where $j = 1, 2, 3$. In the case $G_2 = 0$ we find

$$n_\mu n_\mu = \frac{(c_2 - c_4\hbar^2\omega^2)\hbar^2\omega^2 - G_1}{2G_3}. \tag{28}$$

In real time the frequency spectrum is continuous and (26) is replaced by

$$\omega^2 = -\frac{1}{\hbar^2 c_3}, \tag{29}$$

indicating the homogeneity of the real time vacuum and the energy conservation whenever the Euclidean vacuum is time dependent, $c_3 > 0$.

Consider first the case of

$$c_3 > 0, \quad (c_2 - \hbar^2 c_4\omega^2)\hbar^2\omega^2 > G_1 + 2G_2, \tag{30}$$

and $G_2 \neq 0$. The spatial rotations are represented as $\psi_\alpha \to \Omega_{\alpha,\beta}\psi_\beta$, where $\Omega \in SU(2)$. The corresponding transformations of the diffuson field, $N \to \Omega^\dagger N \Omega$,

$$n_0 + n \cdot \sigma \longrightarrow \Omega^\dagger (n_0 + n \cdot \sigma)\Omega, \tag{31}$$

leave the lagrangian (19) invariant. But the rotational symmetry is broken by the $n \neq 0$ spin term and (31) gives three broken symmetries of the ground vacuum. The fourth broken continuous symmetry is the translation in the time, $\tau \to \tau + \tau_0$. Thus there are four soft modes in this phase which support the long range density fluctuations. Note that the time translation invariance is broken in the imaginary time direction only and is left intact for the real time according to (29). For $G_2 = 0$ the broken symmetry is larger and includes $O(4)$ instead of the rotation group $O(3)$.

In the regions where (30) is not valid one is left with the usual Goldstone mode only which corresponds to the charge conservation and the electron diffusion reflects the features known from the perturbation expansion around the trivial vacuum.

n^2 and ω are the two non-trivial parameters of the vacuum and the constants c_j and G_k of the action appear through these combinations in the tree-level structure. This explains our truncation (18). In fact, the omitted higher order terms can only modify these two parameters without introducing any further tree-level effects. It remains to be seen if the radiative corrections generate further relevant coupling constants in the effective theory.

V. COOPERONS

To support the conducting phase one needs long range current-current correlation functions. We shall identify in this section the coupling constant region of the effective theory for the auxiliary field Q where the inhomogeneous vacuum produces a desired long range dynamics. We start with the effective model $S_Q[Q^*, Q, u] = S_{Q,N}[Q^*, Q, N = 0, u]$ with the truncations $Z_Q = 1$, $M_Q = M$, $U_Q = 0$, and

$$K_Q(z) = r^2 - (z^2 - r)^2 + r^{-2} z (z^2 - r)^2. \tag{32}$$

Since the vacuum will contain a condensate of charges the static, external electrostatic potential $u_{cr}(x)$ of the crystalline structure will be retained, as well.

The distinguishing feature of the conductance in this model is that it appears on the tree level without relying on soft fluctuations. This happens because the cooperon field is charged and the corresponding two-electron states propagate on the periodic background field $u_{cr}(x)$ of the solid state crystal. The higher order derivatives in the time modify the frequency of the saddle point. When the frequency is in a band with extended states of u_{cr} then the saddle point is delocalized. We have an inhomogeneous condensate of charged particles and a new conduction mechanism in this case.

The fluctuating component $v(x, t)$ of the temporal photon field,

$$u(x, t) = u_{cr}(x) + v(x, t) \tag{33}$$

is responsible for the projection into the gauge invariant subspace, identified by Gauss' law. The invariance under the time dependent gauge transformations

$$Q(x,t) \longrightarrow e^{iq\alpha(t)}Q(x,t), \qquad u(x,t) \longrightarrow u(x,t) - \partial_t \alpha, \tag{34}$$

where $q = 2e/\hbar c$, requires that the time derivative always appears in the combination $D_t = \partial_t + iqu$. We expect higher order derivative terms in the action which introduce higher order, non-linear terms in the photon field u. There is no reason to find non-linearity in the external field of the solid state crystal u_{cr}, thus the second equation in (34) will be applied for the fluctuations of the photon field only and the external field is kept gauge invariant according to the background field method [36],

$$u_{cr}(x) \longrightarrow u_{cr}(x), \quad v(x,t) \longrightarrow v(x,t) - \partial_t \alpha. \tag{35}$$

The effective lagrangian in this background gauge is chosen to be

$$\begin{aligned} L_Q = tr\Bigg[& Q^\dagger (i\hbar D_t - c_2 \hbar^2 D_t^2 - c_3 \hbar^3 i D_t^3 + c_4 \hbar^4 D_t^4 + i c_5 \hbar^5 D_t^5) Q \\ & - \frac{\hbar^2}{2M} Q^\dagger \Delta Q - \frac{2e}{c} u_{cr} Q^\dagger Q \Bigg] + \frac{1}{2} (\nabla u)^2. \end{aligned} \tag{36}$$

We look now for the possibility of having a non-vanishing conductivity produced by the two-electron bound states due to a saddle point in the Euclidean theory,

$$Q_{cl}(x,\tau) = e^{i\omega\tau}\chi(x), \qquad v_{cl}(x,\tau) = v. \tag{37}$$

It is worthwhile noting that this saddle point breaks the invariance with respect to the gauge transformation with

$$\alpha(\tau) = \frac{\omega\tau}{q} \tag{38}$$

in (34) which mixes the positive and negative Matsubara modes. The imaginary time equations of motion for χ and v are the equations for the saddle point,

$$0 = \left[\hbar(\partial_\tau + iqv) + c_2 \hbar^2 \left(\partial_\tau^2 + iqv\partial_\tau + iq\partial_\tau v - q^2 v^2 \right) + \cdots + \frac{\hbar^2}{2m}\Delta - u_{cr} \right] Q, \tag{39}$$

and

$$0 = \partial_\tau^2 v - \partial^2 v + \hbar tr Q^* Q + c_2 \hbar^2 tr \left(Q^* \partial_\tau Q - (\partial_\tau Q^*) Q + 2qiv Q^* Q \right) + \cdots, \tag{40}$$

respectively. They are simplified by the ansatz (37) to

$$\begin{aligned} 0 &= \left[K_Q(i\xi) + \frac{\hbar^2}{2m}\Delta - u_{cr} \right] \chi \\ &= \left[r^2 - (\xi^2 + r)^2 + i r^{-2} \xi (\xi^2 + r)^2 + \frac{\hbar^2}{2m}\Delta - u_{cr} \right] \chi, \end{aligned} \tag{41}$$

and

$$
\begin{aligned}
0 &= \frac{d}{d\xi} K_Q(i\xi) \\
&= \frac{d}{d\xi}\left[r^2 - (\xi^2 + r)^2 + ir^{-2}\xi(\xi^2 + r)^2\right],
\end{aligned}
\tag{42}
$$

where $\xi = \omega + qv$. Eq. (42) is solved first for v for a given ω. The result is substituted into the first equation which yields a Schrödinger-like equation,

$$
\left[-\frac{\hbar^2}{2m}\Delta + u_{cr}\right]\chi = K_Q(i\xi)\chi. \tag{43}
$$

This result motivates our particular choice (32). In order for $\chi(x)$ to be extended, $\tilde{E} = K_Q(i\xi)$ must be real. This requires in Euclidean space-time that the odd powers of z cancel in $K_Q(z)$ at the solution of (42). $K_Q(z)$ should contain at least a term $O(z)$, thus we need a polynomial of order 5. The parametrization of $K_Q(z)$ was chosen in such a way that $\xi = \sqrt{-r}$ and the eigenvalue of the Schrödinger equation is real and positive for $r < 0$.

The real time equation of motion for the photon field is

$$
0 = \frac{d}{d\xi} K_Q(\xi). \tag{44}
$$

It has no solution with real ξ for $-(5/4)^{1/3} < r < 0$ indicating the stationarity of the vacuum and the energy conservation in real time in this parameter regime.

We calculate the conductivity by means of the linear response approach. First we introduce a weak external electric field,

$$
u_{cr}(x) \longrightarrow u_{cr}(x) - \epsilon z \tag{45}
$$

and then compute the expectation value of the current operator,

$$
J = \frac{\hbar}{2im} tr\,(Q^* \partial Q - \partial Q^* Q) \tag{46}
$$

in the leading order of the saddle point expansion. In this order the expectation value is given by the saddle point,

$$
\langle Q^* Q\rangle = Q^*_{cl} Q_{cl} + \mathcal{O}(\hbar). \tag{47}
$$

The electric field dependence of the saddle point can be obtained by the help of the Rayleigh-Schrödinger perturbation expansion for (43),

$$
\chi_k = \chi_k^{(0)} + \epsilon \sum_{n \neq k} \frac{\langle \chi_k^{(0)} | z | \chi_n^{(0)} \rangle}{E_n^{(0)} - E_k^{(0)}} \chi_n^{(0)} + \mathcal{O}(\epsilon^2), \tag{48}
$$

where $\chi_n^{(0)}$ are the solutions of (43), by assuming a discrete spectrum, a large but finite quantization volume. The real part of the D.C. conductivity of the level χ_k is then given by

$$
Re\sigma = \frac{\hbar}{m} Im \sum_{n \neq k} \frac{\langle \chi_k^{(0)} | z | \chi_n^{(0)} \rangle}{E_n^{(0)} - E_k^{(0)}} \int dx \chi_k^{(0)*}(x) \partial_z \chi_n^{(0)}(x). \tag{49}
$$

We are now able to discuss the phase structure from the point of view of the conductance. Note that we always have a non-trivial saddle point for $r < 0$, $\xi = +\sqrt{-r}$, $\tilde{E} = r^2$. On the one hand, since u_{cr} is periodic one is always in the continuum of the spectrum of (43) for $\tilde{E} > 0$. On the other hand, the saddle point is trivial for $r > 0$. Thus the long range structure of the background field changes in the vacuum and there is a phase transition at $r = r_0 = 0$.

The eigenvalue in (43) is positive and falls into the continuum for $r < 0$ and we have $K_Q(i\xi) = E_n^{(0)}$ with $n = k$. The wave functions $\chi_n^{(0)}(x)$ with $E_n^{(0)} \approx E_k^{(0)}$ are extended, as well, and (49) is non-vanishing. Since the parameter r comes from an effective theory it is μ dependent. The electron density corresponding to μ_{cr} where $r(\mu_{cr}) = 0$ is the conductor-insulator transition point in this model. Even if the classical conductivity is canceled by the cooperon pole contribution in the usual trivial vacuum for $\mu < \mu_{cr}$ and the electrons are localized, (49) represents a contribution which makes the system conducting when $\mu > \mu_{cr}$.

Note that the conductivity (49) comes from a tree level effect and requires no soft modes though they are present because the vacuum breaks the invariance under spatial rotations and translation of the imaginary time. One may call J a supercurrent in (46) because it corresponds to the particles which make up the condensate. For the usual superfluids the condensate is homogeneous and does not support classical current. In our case the positive energy eigenstate of (43) are scattering states and yield a new contribution to the conductivity when the propagators (47) are used in the Kubo formula. The origin of such a conductivity is the highly populated extended state in Euclidean space-time which is generated by the higher order derivative terms in the effective action and make the hopping between the adjacent ions possible.

VI. SUMMARY

An effective theory was suggested for the description of the conductor-insulator transition in strongly disordered systems along the lines of ref. [18]- [20]. Our starting point is the fact that the main effect of quenched disorder is an effective interaction for the electrons which is highly non-local in time. We assumed that this non-locality can be generated by a quasi-local effective theory for local fields but having few higher order derivatives in time. The highly non-local interactions in time are generated in this model by a “condensation” mechanism, in a vacuum which has extended saddle point structure in imaginary time.

A spatially homogeneous but imaginary time-dependent neutral condensate gives rise to Goldstone modes and a pole in the density-density correlation function in the leading order of the loop expansion. It remains to be seen if this pole contribution is canceled by the higher order radiative corrections as in a homogeneous vacuum [12].

The effective theory supports a space and (imaginary)time dependent vacuum for certain values of the coupling constants. This vacuum is a condensate of charged particle-particle modes in an extended state and the classical current induced by an external electric field is non-vanishing. This provides a new conduction mechanism.

The inhomogeneous saddle points are excluded in 1+1 space-time dimensions and systems in 1 spatial dimensional cannot acquire a conductive phase by this mechanism [37]. But 2 spatial dimensions allow the dynamical breakdown of continuous symmetries and a delocalized phase appears.

The effective theory studied here was chosen in such a manner that its vacuum became inhomogeneous. Further work is clearly needed to decide whether such a rather unusual rearrangement can be justified in certain materials by a detailed derivation of the effective theory from a more fundamental level.

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